11.7

More Than Meets the Eye

Transformations of Quadratic Functions

LEARNING GOALS

In this lesson, you will:

- Translate quadratic functions.
- Reflect quadratic functions.
- Dilate quadratic functions.
- Write equations of quadratic functions given multiple transformations.
- Graph quadratic functions given multiple transformations.
- Identify multiple transformations given equations of quadratic functions.

KEY TERMS

- vertical dilation
- dilation factor

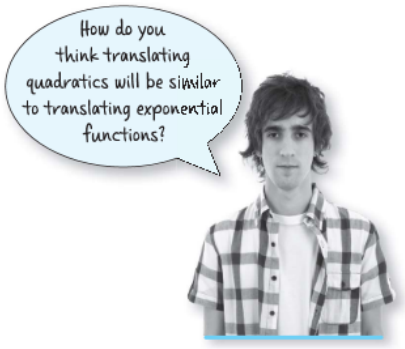
In 1854, Ignazio Porro did some transformational geometry and invented the "Porro Prism."

When an image is gathered by a traditional lens, it is upside down. Using right triangular prisms, Porro was able to turn the image upright.

There is a connection between Porro and binoculars too!

PROBLEM 1 Translations and Reflections

1. Consider the three quadratic functions shown, where $g(x)$ is the basic function.
   - $g(x) = x^2$
   - $c(x) = x^2 + 3$
   - $d(x) = x^2 - 3$



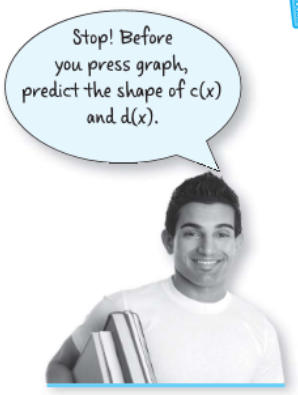
a. Write the functions $c(x)$ and $d(x)$ in terms of the basic function $g(x)$. For each, determine whether an operation is performed on the *function* $g(x)$ or on the *argument* of the function $g(x)$. Describe the operation.

$c(x) =$ _____

$d(x) =$ _____

11

b. Use a graphing calculator to graph each function with the bounds $[-10, 10] \times [-10, 10]$. Then, sketch the graph of each function on the coordinate plane provided. Label each graph.



c. Compare the graphs of $c(x)$ and $d(x)$ to the graph of the basic function. What do you notice?

d. Describe the type of transformation performed on $g(x)$ to result in:

- $c(x)$

- $d(x)$

Vertical translations are performed on $g(x)$ to result in $c(x)$ and $d(x)$.

e. Use coordinate notation to represent the vertical translation of each function.

- $c(x) = x^2 + 3$

  $(x, y) \rightarrow$ _____

- $d(x) = x^2 - 3$

  $(x, y) \rightarrow$ _____

f. Write $c(x)$ and $d(x)$ in vertex form, $y - a(x - h)^2 + k$.

$c(x) -$ _____

$d(x) -$ _____

2. Consider the three quadratic functions shown, where $g(x)$ is the basic function.

- $g(x) = x^2$
- $j(x) = (x + 3)^2$
- $k(x) = (x - 3)^2$

a. Write the functions $j(x)$ and $k(x)$ in terms of the basic function $g(x)$. For each, determine whether an operation is performed on the *function* $g(x)$ or on the *argument* of the function $g(x)$. Describe the operation.

$j(x) =$ _____

$k(x) -$ _____

b. Use a graphing calculator to graph each function with the bounds $[-10, 10] \times [-10, 10]$. Then, sketch the graph of each function on the coordinate plane provided. Label each graph.

c. Compare the graphs of $j(x)$ and $k(x)$ to the graph of the basic function. What do you notice?

11

d. Describe the type of transformation performed on $g(x)$ to result in:

- $j(x)$
- $k(x)$

Horizontal translations are performed on $g(x)$ to result in both $j(x)$ and $k(x)$.

e. Use coordinate notation to represent the horizontal translation of each function.

- $j(x) = (x + 3)^2$

  $(x, y) \rightarrow$ _____
- $k(x) = (x - 3)^2$

  $(x, y) \rightarrow$ _____

f. Write $j(x)$ and $k(x)$ in vertex form.

$j(x) =$ _____

$k(x)$ _____

3. Consider the three quadratic functions shown, where $g(x)$ is the basic function.

- $g(x) = x^2$
- $m(x) = -x^2$
- $n(x) = (-x)^2$

a. Write the functions $m(x)$ and $n(x)$ in terms of the basic function $g(x)$. For each, determine whether an operation is performed on the *function* $g(x)$ or on the *argument* of the function $g(x)$. Describe the operation.

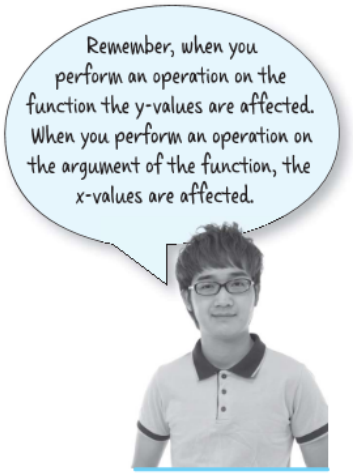
$m(x) =$ _____

$n(x) =$ _____

b. Use a graphing calculator to graph each function with the bounds $[-10, 10] \times [-10, 10]$. Then, sketch the graph of each function on the coordinate plane provided. Label each graph.

11

c. Compare the graphs of $m(x)$ and $n(x)$ to the graph of the basic function. What do you notice?



d. Describe the type of transformation performed on $g(x)$ to result in:

- $m(x)$

- $n(x)$

Reflections are performed on $g(x)$ to result in both $m(x)$ and $n(x)$.

e. Use coordinate notation to represent the reflections of each function.

- $m(x) = -x^2$

  $(x, y) \rightarrow$ _____

- $n(x) = (-x)^2$

  $(x, y) \rightarrow$ _____

f. What is special about a reflection of $g(x)$ over the vertical line $x = 0$? Explain your reasoning.

g. Write $m(x)$ and $n(x)$ in vertex form.

$m(x)$ _____

$n(x)$ – _____

11

4. Describe each graph in relation to its basic function.

a. Compare $f(x) = x^2 + b$ when $b > 0$ to the basic function $g(x) = x^2$.

b. Compare $f(x) = x^2 + b$ when $b < 0$ to the basic function $g(x) = x^2$.

c. Compare $f(x) = (x + b)^2$ when $b > 0$ to the basic function $g(x) = x^2$.

d. Compare $f(x) = (x + b)^2$ when $b < 0$ to the basic function $g(x) = x^2$.

e. Compare $f(x) = -x^2$ to the basic function $g(x) = x^2$.

f. Compare $f(x) = (-x)^2$ to the basic function $g(x) = x^2$.

PROBLEM 2 Dilation Station

1. Consider the three quadratic functions shown, where $g(x) = x^2$ is the basic function.

- $g(x) = x^2$
- $p(x) = 3x^2$
- $q(x) = \frac{1}{3}x^2$

a. Write the functions $p(x)$ and $q(x)$ in terms of the basic function $g(x)$. For each, determine whether an operation is performed on the *function* $g(x)$ or on the *argument* of the function $g(x)$. Describe the operation.

$p(x) =$ _____

$q(x) =$ _____

b. Use a graphing calculator to graph each function with the bounds $[-10, 10] \times [-10, 10]$. Then, sketch the graph of each function on the coordinate plane provided. Label each graph.

11

c. Compare the graphs of $p(x)$ and $q(x)$ to the graph of the basic function. What do you notice?

d. Complete the table of ordered pairs for the three given functions.

$g(x) = x^2$	$p(x) = 3x^2$	$q(x) = \frac{1}{3}x^2$
(−2, 4)	(−2, __)	(−2, __)
(−1, 1)	(−1, __)	(−2, __)
(0, 0)	(0, __)	(0, __)
(1, 1)	(1, __)	(−2, __)
(2, 4)	(2, __)	(−2, __)

e. Use your table to compare the ordered pairs of the graphs of $p(x)$ and $q(x)$ to the ordered pairs of the graph of the basic function $g(x)$. What do you notice?

11

A **vertical dilation** of a function is a transformation in which the y-coordinate of every point on the graph of the function is multiplied by a common factor called the **dilation factor**. A vertical dilation stretches or shrinks the graph of a function.

You can use the coordinate notation shown to indicate a vertical dilation.

$(x, y) \rightarrow (x, ay)$, where a is the dilation factor.

f. Use coordinate notation to represent the vertical dilation of each function.

- $p(x) = 3x^2$

  $(x, y) \rightarrow$ _____

- $q(x) = \frac{1}{3}x^2$

  $(x, y) \rightarrow$ _____

2. Describe each graph in relation to its basic function.

   a. Compare $f(x) = ax^2$ when $a > 1$ to the basic function $g(x) = x^2$.

   b. Compare $f(x) = ax^2$ when $0 < a < 1$ to the basic function $g(x) = x^2$.

PROBLEM 3 Name That Parabola

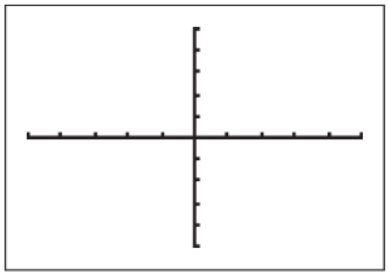
1. Use the given characteristics to write a function and sketch a graph of $f(x)$.

   a. Write a function in vertex form and sketch a graph that has these characteristics:

   - The function is quadratic.
   - The function is continuous.
   - The parabola opens upward.
   - The function is translated 5 units to the right of $f(x) = x^2$.

11

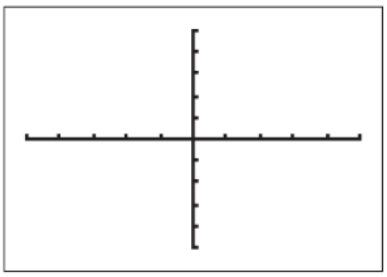
Equation: $f(x) =$ _____



b. Write a function in vertex form and sketch a graph that has these characteristics:

- The function is quadratic.
- The function is continuous.
- The parabola opens downward.
- The function is translated 1 unit down from $f(x) = -x^2$ and is vertically dilated with a dilation factor of 2.

Equation: $f(x) =$ _____

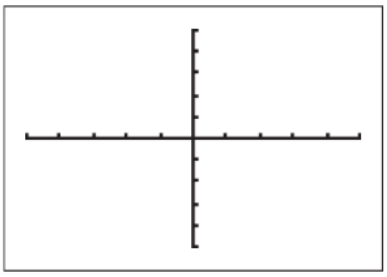


c. Write a function in vertex form and sketch a graph that has these characteristics:

11

- The function is quadratic.
- The function is continuous.
- The parabola opens upward.
- The function is translated 4 units down and 3 units to the left of $f(x) = x^2$.
- The function is vertically dilated with a dilation factor of $\frac{1}{4}$.

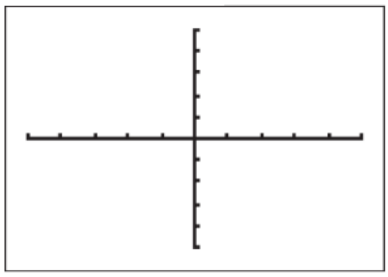
Equation: $f(x) =$ _____



d. Write a function in vertex form and sketch a graph that has these characteristics:

- The function is quadratic.
- The function is continuous.
- The parabola opens downward.
- The function is translated 8 units up and 2 units to the right of $f(x) = x^2$.

Equation: $f(x) =$ _____



2. Based on the equation of each function, describe how the graph of each function compares to the graph of $g(x) = x^2$.

a. $w(x) = (x + 2)^2$

b. $t(x) = 3x^2 + 4$

c. $z(x) = -(x - 1)^2 - 10$

d. $r(x) = \frac{1}{2}(x + 6)^2 + 7$

Be prepared to share your solutions and methods.